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A Model for Flow of Blood in an Equally Branched Channel

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Abstract

A mathematical model has been developed for studying the characteristics of blood flow in slip-flow regime through a bifurcated artery under the action of a static transverse magnetic field. The model has been analysed to find the effects of slip parameter as well as Hartmann number on the velocity profile, pressure gradient and skin friction. Analytical expressions are obtained by using momentum integral equation technique and are also depicted through graphs.

Key words : Bifurcation, atherosclerotic plaques, intimal cushions, aneurysm, skin friction, slip parameter.

Introduction

It is observed that local disturbances and high shear forces cause certain cardiovascular lessions like atherosclerotic plaques, intimal cushions etc. near bifurcation. Aneurysm also occurs in brain circulatory system at and near the apex of bifurcation. Since blood is an electrically conducting fluid (1938), the MHD principles may be used to regulate the flow of blood in arterial systems. In 1936, Kolin introduced the idea of electromagnetic fields in medical sciences. It was shown by Korchevskii and Marochunik (1965) that external magnetic fields can be used in the treatment of some kinds of diseases like cardiovascular diseases and in the diseases with accelerated blood circulation such as haemorrhages and hypertention etc. Several researchers (1973, 1974, 1978) studied the effects of magnetic field on the flow of blood theoretically taking different kinds of models. Number of investigators discussed the matter of bifurcation mathematically (1999, 2000, 2003, 2008, 2006, 2000), considering various types of external effects on human arterial system. Suri and Suri (1981) discussed the charecteristics of blood flow near bifurcation in presence of static magnetic field.

Most of the aformentioned works are based on the usual assumption of the no-slip condition at the wall of the blood vessel. Bugliarello and Hayden (1962) and Bennet (1967) suggested that there might be possibility of existence of slip-velocity for red cells at the vessel wall on the basis of their invitro experiments, which also concluded that the slip condition was real and not hypothetical.

In the present analysis, it is proposed to develop a mathematical model to study the characteristics of blood flow through an equally branched artery in presence of static magnetic field and by accounting for the slip at the endothelium of the blood vessel. Momentum integral technique has been employed to solve the problem. Analytical expressions for the blood velocity, pressure gradient and skin friction are obtained and the slip parameter has been found to effect largely on those quantities which are shown through graphs.

Mathematical Formulation

To describe the bifurcation model in an artery we consider a two-dimensional, non-conducting parallel plates with equally branched channel, such

that one stream of blood from trunk is branched into two different streams. The effect of static magnetic field is considered in this model which is applied in a direction perpendicular to the flow of blood. The thickness of the bifurcating wall is taken as of negligibly small magnitude. In both the inner and outer walls of the bifurcated artery, slip parameter is taken to describe the phenomenon of slip velocity at the wall. In figure 1 the geometry of the bifurcation model is shown and is taken as same as (1973). Here blood is supposed to be Newtonian, incompressible, homogeneous and viscous fluid. Since the Reynolds number does not vary much in the region of any one bifurcation, the viscosity of blood is considered to be constant.

Considering u and v as velocity components in the directions of x and y axis respectively at time t in the flow field, we may write the two-dimensional boundary layer equations in presence of transverse static magnetic field as

$$\frac{\partial u}{\partial t}+\frac{1}{\rho}\frac{dp}{dx}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho} \qquad (1)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (2)$$

In the above equations σ, ρ and μ denotes the electrical conductivity, density and viscosity of blood respectively while p and B_0 represents pressure and applied magnetic field.

We introduce the following non-dimensional quantities as

$$\left.\begin{aligned} &x'=\frac{x}{b},\ u'=\frac{u}{m/2b\rho},\ h(x,t)=\frac{\frac{dp}{dx}}{\mu m/2b^3\rho},\ y'=\frac{y}{b},\\ &v'=\frac{v}{m/2b\rho},\ R=\frac{m}{2\mu},\ t'=\frac{t}{b^2\rho/\mu},\ \text{and}\ M=bB_0\sqrt{\sigma/\mu}\end{aligned}\right\} \qquad (3)$$

Using (3) in (1) and (2) we may write these equations as (after dropping the primes)

$$\frac{\partial u}{\partial t}+h=\frac{\partial^2 u}{\partial y^2}-M^2 u \qquad (4)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (5)$$

The initial and boundary conditions are taken as

(i)Initialy at

$$t=0;\ u=u_0 \text{ for all } y \qquad (6)$$

(ii)The boundary conditions, considering the effect of slip parameter on the inner wall of the trunk and branches, are given by

$$x<0;\ u+\alpha\frac{\partial u}{\partial y}=0,\ v=0 \text{ for } y=\pm 1 \qquad (7)$$

and

$$x\geq 0;\ u+\alpha\frac{\partial u}{\partial y}=0,\ v=0 \text{ for } y=0,\pm 1 \qquad (8)$$

(iii)The law of conservation of mass is preserved, i.e.,

$$\int_{-h}^{h}\rho u dy=m \qquad (9)$$

which can be written in non-dimensional form as

$$\int_{-1}^{1}u dy=1 \qquad (10)$$

due to symmetry about $y=0$.

Solutions

The problem is solved by using momentum integral technique. Here it is noted that the pressure gradient parameter h is unknown and the boundary layer solutions of (4) and (5) do not satisfy the boundary conditions on $y=\pm 1$ in the branches. Thus it is quite justified to analyse this problem with the help of momentum integral technique. As the flow pattern is symmetrical about $y=0$, the analysis is done for only one of the branches, i.e., taking the region bounded by

$$\left.\begin{array}{l} 0 \le x < +\infty \\ 0 \le y \le +1 \end{array}\right\} \qquad (11)$$

Taking Laplace transform of (4) we obtain the momentum equation as

$$s\bar{u} - u_0 + \bar{h} = \frac{\partial^2 u}{\partial y^2} - M^2\bar{u} \qquad (12)$$

where $\bar{u}(s) = \int_0^\infty e^{-st} f(t)\,dt$.

The moment of momentum equation is given as

$$ys\bar{u} - yu_0 + y\bar{h} = y\frac{\partial^2 u}{\partial y^2} - yM^2\bar{u} \qquad (13)$$

We obtain the integral form of the boundary layer equations by integrating (12) and (13) from $y = 0$ to $y = 1$ as

$$\int_0^1 (s + M^2)\bar{u}dy - \int_0^1 u_0 dy + \int_0^1 \bar{h}dy = \int_0^1 \frac{\partial^2 u}{\partial y^2} dy \qquad (14)$$

$$\int_0^1 y(s + M^2)\bar{u}dy - \int_0^1 yu_0 dy + \int_0^1 y\bar{h}dy = \int_0^1 y\frac{\partial^2 u}{\partial y^2} dy \qquad (15)$$

We may write

$$\int_0^1 \frac{\partial^2 \bar{u}}{\partial y^2} dy = \left(\frac{\partial \bar{u}}{\partial y}\right)_{y=1} - \left(\frac{\partial \bar{u}}{\partial y}\right)_{y=0} \qquad (16)$$

$$\int_0^1 y\frac{\partial^2 \bar{u}}{\partial y^2} dy = (1+\alpha)\left(\frac{\partial \bar{u}}{\partial y}\right)_{y=1} - \alpha\left(\frac{\partial \bar{u}}{\partial y}\right)_{y=0} \qquad (17)$$

and using these results, equations (14) and (15) become

$$(s + M^2)\int_0^1 \bar{u}dy - \int_0^1 u_0 dy + \bar{h} = \left(\frac{\partial \bar{u}}{\partial y}\right)_{y=1} - \left(\frac{\partial \bar{u}}{\partial y}\right)_{y=0} \qquad (18)$$

$$(s + M^2)\int_0^1 y\bar{u}dy - \int_0^1 yu_0 dy + \frac{1}{2}\bar{h} = (1+\alpha)\left(\frac{\partial \bar{u}}{\partial y}\right)_{y=1} - \alpha\left(\frac{\partial \bar{u}}{\partial y}\right)_{y=0} \qquad (19)$$

To solve (18) and (19) we assume u as a polynomial in y with coefficients as functions of x and t in a manner given below

$$u(x, y, t) = f_0(x,t) + yf_1(x,t) + y^2 f_2(x,t) + y^3 g(x,t) \qquad (20)$$

in which f_0, f_1, f_2 and g are arbitrary functions of and .

Laplace transform of (20) gives (21)

Using Laplace transform on (8) and (10), we get, by using (21), we get the values of , , in terms of as (22)

Thus equation (21) then becomes (23)

Substituting (23) in (18) and (19) we get (24)

in which is the value of at .

Applying inverse Laplace transform in equations (23) and (24) we obtain (25)

(26)

from equation (5) we may write (27)

The shear stress parameter at the inner wall and at the outer wall are obtained from equation (25), as (28)

(29)

The blockage effect is defined as the value of the pressure gradient at the fork section and is determined from equations (3) and (26) as (30)

An important observation is that when there is no slip at the wall, then and then all the above results agree with (1981).

Numerical results and discussions

After obtaining the expressions of velocity profile and shear stress, the influences of various parameters on them are shown through graphs. The effect of slip parameter α and Hartmann number M is specially illuminated here.

In figure 2 the influence of in velocity profile with respect to is shown, taking specific values of , and . It is observed here that the value of increases with the increase in for ; whereas the situation is totally opposite whenever i.e., the value of decreases with the increase of . Figure 3 is constructed showing the effect of Hartmann number on velocity profile with respect to , taking specific values of , and . As the value of increases the value of decreases for . Again as the value of increases the value of also increases for . In figure 4 the effect of on velocity profile with respect to is shown, taking fixed values of , and . It is noticed in this figure that the value of increases with the increase in the value of for , whereas the value of decreases with the increase in the value of for .

Figure 5 is drawn to show the effect of slip parameter on velocity profile with respect to time , taking fixed values of , and . In this figure it is observed that as the value of increases, the initial values of decreases but after a certain value of the value of decreases rapidly and after the value of increases with the increasing value of . Figure 6 is constructed showing the effect of on velocity profile with respect to Hartmann number . It is observed in this figure that the value of increases with the increase in the value of .

In figure 7 the nature of shear stress at the inner wall is depicted with respect to time . Here the value of and are taken fixed and it is observed that the value of increases with the increase in the value of . Figure 8 is drawn to show the nature of shear stress at the outer wall with respect to time . The values of and are taken as fixed. In this figure it is noted that as the value of increases, the initial value of decreases; but whenever the value of increases with the increasing value of . In both the cases it is noticed that the values of and decreases drastically with increasing value of time .

Conclusion

It is obvious from the above analysis that the slip parameter influences largely to regulate the velocity of blood flow in human arteries. It also has effects on shear stress in both the inner and outer walls of arteries. The effects of Hartmann number and time on flow pattern are also discussed here. So the present discussion helps us to determine the nature of flow pattern of blood as well as to regulate this flow pattern and shear stress by making suitable adjustments on external magnetic field.

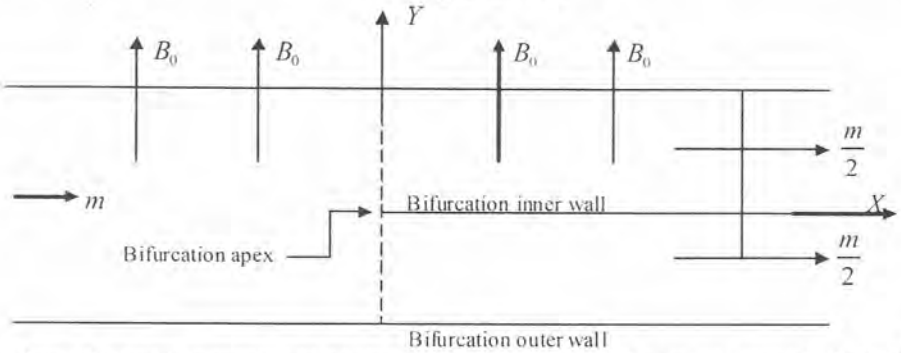

Fig. 1 : Unsteady flow of blood in a parallel plate channel in the presence of external magnetic field B_0 .

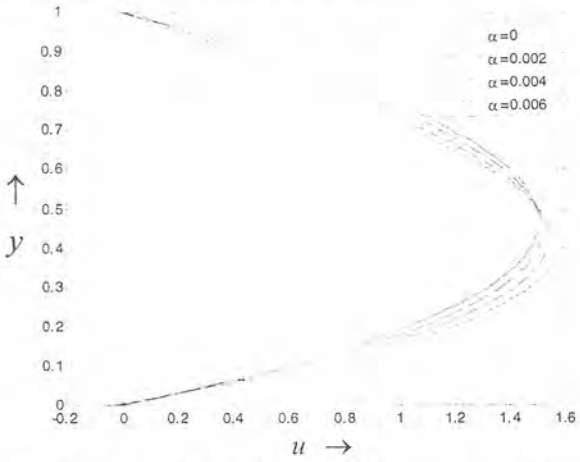

Fig. 2 : Velocity profile for different values of α taking $g_0 = 50, M = 10, t = 0.05$.

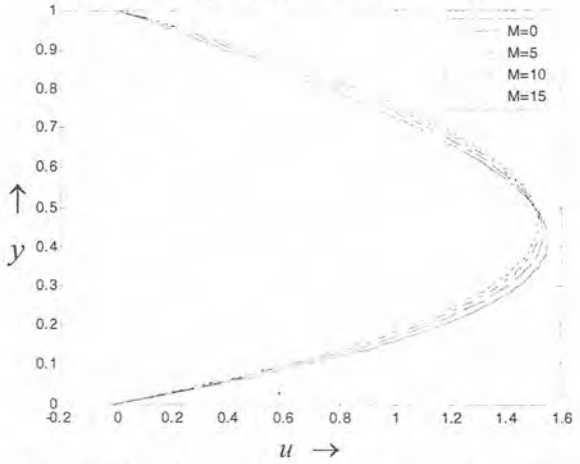

Fig. 3 : Velocity profile for different values of M taking $g_0 = 50, \alpha = 0.002, t = 0.05$.

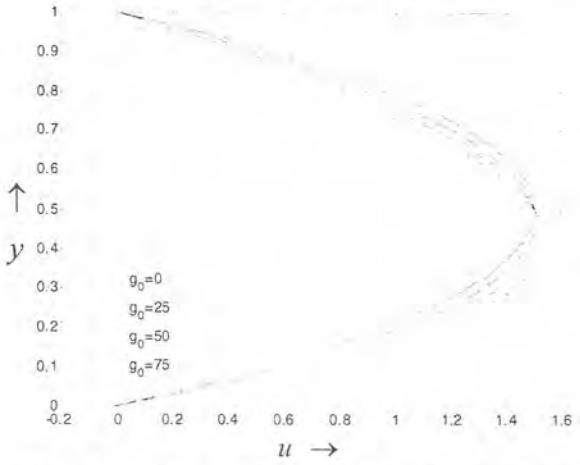

Fig. 4 : Velocity profile for different values of g_0 taking $M = 10, \alpha = 0.002, t = 0.05$.

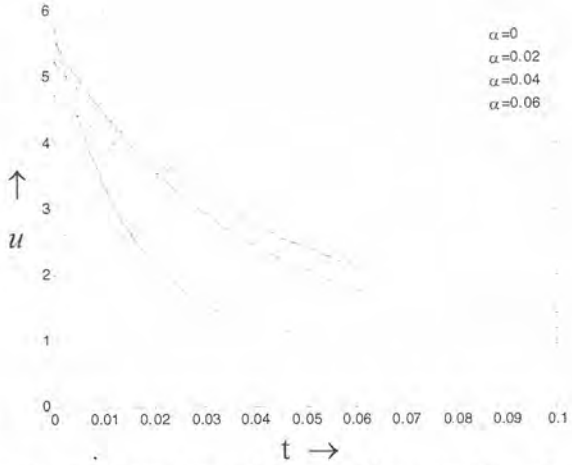

Fig. 5 : Velocity profile for different values of α taking $M = 10, g_0 = 100, y = 0.2$.

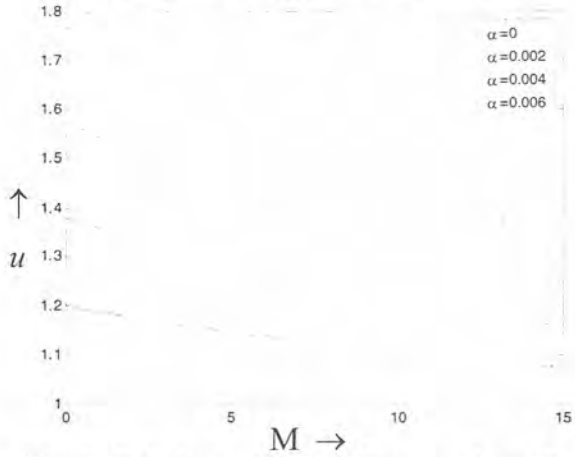

Fig. 6 : Velocity profile for different values of α taking $t = 0.05, g_0 = 100, y = 0.2$.

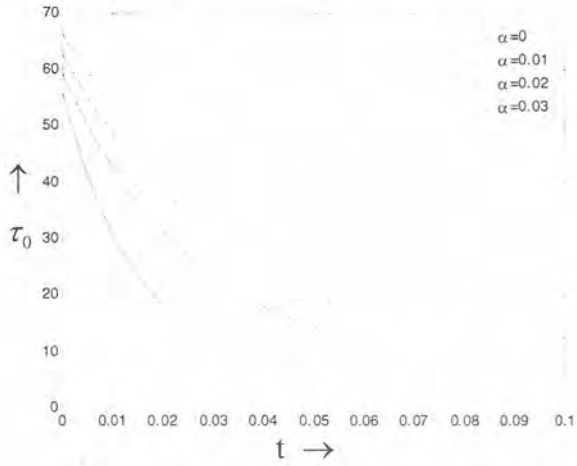

Fig. 7 : Shear stress τ_0 for different values of α taking $M = 10, g_0 = 100$.

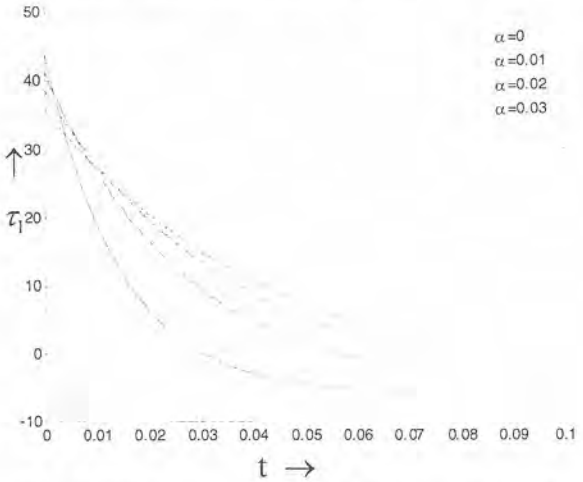

Fig. 8 : Shear stress τ_1 for different values of α taking $M = 10, g_0 = 100$.

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